P425/1
PURE
MATHEMATICS
Paper 1
July/Aug. 2022
3 hours

ASK INTERGRATED TEACHERS' MOCK EXAMINATIONS BURREAU

AITEL JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 Hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in the section* **A** *and*

Answer any **five** *questions in section* **B**

 Turn Over

SECTION A (40 MARKS)

1. The roots of a quadratic equation $x^2 + (7+p)x + p = 0$ are ∝ and β.
   Given that ∝and β differ by 5, find the possible values of p. (5 marks)

2. Evaluate $\int_2^3 \frac{2}{x^2 - 4x + 13} dx$ (5 marks)

3. Show that the parametric equations $x = 5 + \frac{\sqrt{3}}{2}\cos\theta \quad y = -3 + \frac{\sqrt{3}}{2}\sin\theta$ represent a circle. Find the radius and centre of the circle. (5 marks)

4. Solve the equation: $3 + 2\cos^2\theta + \tan\theta = 4\cos^2\theta$, for $0^0 \leq \theta \leq 360^0$ (5 marks)

5. How many team of 6 players can be formed from a group of 7 boys and 5 girls if
   (i) Each team should have at least 3 boys and a girl
   (ii) Each team contains at most 3 girls (5 marks)

6. Use raw Echelon reduction to solve the following Equations simultaneously.
   (i) $3x = z - 2y$
   (ii) $3y = x + 2z + 1$
   (iii) $3z = 2x - 2y + 3$ (5 marks)

7. Differentiate $y = x\ln x$ from first principles . (5 marks)

8. Find the equation of the normal to the curve $x^2\tan x - xy - 2y^2 = -2$ at the point (0,1) (5 marks)

SECTION B (60 MARKS)

Answer any **five** *questions*

9. (a) Show that the lines $r = 5 + 3 - 5k + u(+2j - 3k)$ and
   $\frac{x-7}{3} = \frac{y+1}{-2} = \frac{z+4}{-2}$ intersect and hence find the coordinates of the point of intersection. (7 marks)

   (b) A plane is at a distance of $\sqrt{11}$ units from the origin. If theline passing through the points A(4,-9,3) and B(6,-7,9) is perpendicular to the plane, find the cartesian equation of the plane. (5 marks)

10. (a) Show that sin3θ= 3sinθ – 4sin$^3\theta$ hence solve the equation $8x^3 - 6x - 1 = 0$
    (7 marks)

    (b) By expressing 5cosx + 8sinx in the form $Rcos(x + \beta)$, where R is a constant and β is an acute angle, solve $5cosx + 8sinx = 7$
    for $0^0 \leq$ x $\leq 360^0$ (5 marks)

11. Evaluate $\int_2^3 \frac{2x+1}{(x-2)(x+1)^2}dx$. Give your answer correct to 3 decimal places.

(12 marks)

12. (a) Prove by induction that $4^{(n+3)} - 3n - 10$ is divisible by 3 for all positive integral values of *n*. (6 marks)

(b) Use De Moivres theorem to prove that the complex number $(\sqrt{3}+i)^n + (\sqrt{3}-i)^n$ is always real and hence find the value of the expression when n = 6. (6 marks)

13. (a) Use the binomial theorem to show that

$$(\sqrt{1+2x}+\sqrt{1-4x})^2 = (2 - x - \frac{5}{2}x^2 + ...)^2$$ (7 marks)

(b) Taking $x = \frac{1}{16}$ use the equation in (a) above to estimate $\sqrt{6}$ to 2 decimal places.

(5 marks)

14. (a) Solve for x in; $9^x - 3^{(x+1)} = 10$ (5 marks)

(b) Solve the following pair of simultaneous equations.

$$\log_2 x^2 + \log_2 y^3 = 1$$
$$\log_2 x - \log_2 y^2 = 4$$
(7 marks)

15. (i) If the curve $y = \frac{x^2 - 4x + 4}{x+1}$ Show that the curve is restricted, state the region and hence investigate the nature of the turning points.

(ii) Determine the equations of asymptotes to the curve

(iii) Sketch the curve

(12 marks)

16. (a) Solve the differential equation.

$y\cos^2 x\frac{dy}{dx} = \tan x + 2$, given that y = 0 when $x = \frac{\pi}{4}$ (5 marks)

(b) A spherical bubble evaporates at a rate proportional to its surface area. If half of it evaporates in 2 hours, when will the bubble disappear? (7 marks)

END